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On an Asymptotic Case of Plane Turbulent Motion of a Weakly Conducting Rotating Fluid in the Presence of a Magnetic Field

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SUMMARY
The final period of decay of homogeneous turbulence in a weakly conducting rotating fluid in the presence of a magnetic field is investigated. By asymptotic expansion techniques, the expressions of the kinetic and magnetic energy tensors are established. After a first stage similar to the non-rotating case, it is shown that modulated plane turbulence occurs when both the applied magnetic field and the axis of rotation are parallel.

1. Introduction

In the last period of decay of magnetodynamic turbulence, the Reynolds numbers are very low. The inertial forces are negligible compared with the viscous forces; hence the quadratic terms may be neglected in the equations of motion which are consequently linear.

The governing equations for an incompressible homogeneous rotating fluid in the presence of a uniform magnetic field were established in 1955 by Lehnert [1] who derived the expression of the kinetic and magnetic energy densities in wave number space. The law of decay of the total energy can be obtained by integration of the energy densities over the whole wave number space, which requests the knowledge of the initial energy distribution. Assuming initial isotropic turbulence and zero magnetic initial fluctuations, Deissler [2], in 1963, calculated numerically the energy decay for different ratios of the kinematic and magnetic viscosities $n = \nu/\lambda$ in a non-rotating fluid. Anisotropic initial conditions for the kinetic energy have been established by Batchelor and Proudman [3] in 1956 and extended in 1963 to the magnetic case by Alexandrou [4] who found analytically a law of decay in $t^{-\frac{5}{2}}$ for the kinetic and magnetic total energies in a non-rotating fluid with $n = 1$. The general case was solved by Nihoul [5] in 1965. He found that there were two contributions to the total energies: one decays asymptotically as $t^{-\frac{5}{2}}$, leads to equipartition of energy between the magnetic and kinetic fields and dominates for n nearly equal to unity; the other one decays asymptotically as t^{-3}, leads to a partition of energy between the two fields in the inverse ratio of the diffusivities and dominates for n much larger or smaller than unity. The t^{-3} component was directly found by Nihoul [6] in 1967 with Golitsyn's simplified equations [7] for low magnetic Reynolds number.

In this paper, the case of a weakly conducting rotating fluid is investigated. Golitsyn's equations are adopted and it is assumed that the applied magnetic field $\boldsymbol{B}$ and the axis of rotation are parallel. Asymptotic expansion techniques are used to obtain each component of the energy tensors. The anisotropic initial conditions established in 1967 by Saffman [8] are chosen for simplicity. The results are easily adapted to Batchelor and Proudman's conditions.

It is found that the energy decay passes by two successive asymptotic stages. In the first one, all elements of the energy tensors decrease as t^{-2} whereas the second stage is strongly anisotropic. All tensor components decay as t^{-2} except the exchange terms between the energy in the direction of both the rotation axis and the applied magnetic field $\boldsymbol{B}$ and the energy contained in the perpendicular plane which decay as t^{-3} and hence become rapidly negligible. It is also established that the kinetic and magnetic energies are both equally shared between

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the $\boldsymbol{B}$-direction and the perpendicular ones. This indicates the existence of an asymptotic plane turbulent motion modulated perpendicularly with the same energy. The appearance of the short transition period between the two regimes is a function of the parameters; it is shown that both successive stages could be observed in experiments with mercury.

2. The Basic Equations

The equations of motion of an incompressible homogeneous fluid rotating with a constant angular velocity $\boldsymbol{\omega}$ can be written in MKS units [1]:

$$\frac{\partial \boldsymbol{v}}{\partial t} + \boldsymbol{v}\cdot\nabla\boldsymbol{v} = 2\boldsymbol{v}\times\boldsymbol{\omega} + \nu\nabla^2\boldsymbol{v} + \boldsymbol{h}\cdot\nabla\boldsymbol{h} - \nabla\varphi \tag{1}$$

$$\frac{\partial \boldsymbol{h}}{\partial t} = \boldsymbol{h}\cdot\nabla\boldsymbol{v} - \boldsymbol{v}\cdot\nabla\boldsymbol{h} + \lambda\nabla^2\boldsymbol{h}\,. \tag{2}$$

$$\nabla\cdot\boldsymbol{v} = 0 \qquad \nabla\cdot\boldsymbol{h} = 0 \qquad (3)\quad(4)$$

where the Local Alfvén velocity $\boldsymbol{h}$ is defined by

$$\boldsymbol{h} = (\mu\rho)^{-\frac{1}{2}}\boldsymbol{B} \tag{5}$$

and where

$$\lambda = (\mu\sigma)^{-1} \tag{6}$$

$$\varphi = \frac{p}{\rho} + \varphi_g + \varphi_c + \tfrac{1}{2}\boldsymbol{h}\cdot\boldsymbol{h} \tag{7}$$

$\boldsymbol{v}$ is the velocity, $\boldsymbol{B}$ the magnetic field, μ the permeability, ρ the density, ν and λ respectively the kinematic and magnetic viscosity, σ the conductivity, p the pressure, φ_g and φ_c the gravific and centrifugal potentials, $\boldsymbol{r}$ the position vector. The magnetic field is assumed to be the sum of a uniform ambient field and an induced fluctuating field, i.e.:

$$\boldsymbol{h}(\boldsymbol{r}, t) = \boldsymbol{b}_o + \boldsymbol{b}(\boldsymbol{r}, t)\,. \tag{8}$$

In the last period of decay of turbulence, the non-linear term may be neglected as compared with the viscous term. The conditions of validity of this simplification may be written

$$R = \frac{vL}{\nu} \ll 1 \tag{9}$$

$$R_m = \frac{vL}{\lambda} \ll 1 \tag{10}$$

where v and L are characteristic scales of the velocity and length. Let us also define b_o and b as characteristic scales of the applied and induced Alfvén velocity.

The Golitsyn's hypothesis [7] allows further simplifications of equation (2). By analogy between equation (2) and the heat conduction equation in a solid body with a diffusivity λ and a source distribution, $\boldsymbol{h}\cdot\nabla\boldsymbol{v} - \boldsymbol{v}\cdot\nabla\boldsymbol{h}$, the accommodation time of the field $\boldsymbol{h}$ to a fluctuation of the source can be estimated to be of order $\lambda^{-1}L^2$. Assuming $\tau \gg \lambda^{-1}L^2$, where τ is the characteristic time of modification of the turbulent field, we may neglect $\partial\boldsymbol{b}/\partial t$ as compared with $\lambda\nabla^2\boldsymbol{b}$. The comparison of the order of magnitude of the subsisting terms indicates that $b \sim O(R_m b_o)$. Consequently the induced magnetic field may be neglected as compared with the applied one and the governing equations reduce to:

$$\frac{\partial \boldsymbol{v}}{\partial t} = 2\boldsymbol{v}\times\boldsymbol{\omega} + \nu\nabla^2\boldsymbol{v} + \boldsymbol{b}_o\cdot\nabla\boldsymbol{b} - \nabla\varphi \tag{11}$$

$$\boldsymbol{b}_o\cdot\nabla\boldsymbol{v} + \lambda\nabla^2\boldsymbol{b} = 0\,. \tag{12}$$

In fact, Golitsyn assumed $\tau \sim O(v^{-1} L)$ which leads to equation (12) under the only hypothesis (10). Nihoul [7] proved that this is not generally true, especially in the last period of decay where $\tau \sim O(v^{-1} L^2)$; he showed that two additional conditions were requested for the validity of Golitsyn's equations:

$$\frac{v}{\lambda} \ll 1 . \tag{13}$$

$$\frac{\lambda^{-1} L^2}{\dfrac{vL}{b_o b}} \sim O\left(\frac{b_o^2 R_m^2}{v^2}\right) \ll 1 . \tag{14}$$

Let us mention that these conditions are verified in laboratory experiment with mercury [12]; hence Golitsyn's hypotheses are not restrictive in this case.

Defining the Fourier transforms* of $\boldsymbol{v}$, $\boldsymbol{b}$, φ by

$$V_i(\boldsymbol{k}, t) = \int v_i(\boldsymbol{r}, t)\, \mathrm{e}^{-i\boldsymbol{kr}} \mathrm{d}\boldsymbol{r}$$

$$B_i(\boldsymbol{k}, t) = \int b_i(\boldsymbol{r}, t)\, \mathrm{e}^{-i\boldsymbol{kr}} \mathrm{d}\boldsymbol{r}$$

$$\Phi(\boldsymbol{k}, t) = \int \varphi(\boldsymbol{r}, t)\, \mathrm{e}^{-i\boldsymbol{kr}} \mathrm{d}\boldsymbol{r} ,$$

equations (11), (12), (3) and (4) can be written,

$$\frac{\partial V_i}{\partial t} = i\, k_k b_{ok} B_i - v k^2 V_i + 2\varepsilon_{ilm} \omega_m V_l - i\, k_i \Phi , \tag{15}$$

$$i\, k_k b_{ok} V_i - \lambda k^2 B_i = 0 , \tag{16}$$

$$k_i V_i = 0 , \tag{17}$$

$$k_i B_i = 0 . \tag{18}$$

where ε_{ilm} is the usual Poisson symbol. Eliminating B_i, equations (15) and (16) reduce to

$$D\boldsymbol{V} = \mathbf{A}\boldsymbol{V} - i\boldsymbol{k}\Phi \tag{19}$$

where the operator D is defined by

$$D \equiv \frac{\partial}{\partial t} + \frac{(k_k b_{ok})^2}{\lambda k^2} + v k^2 , \tag{20}$$

and the tensor $\mathbf{A}$ by:

$$\mathbf{A} \equiv 2 \begin{pmatrix} 0 & \omega_3 & -\omega_2 \\ -\omega_3 & 0 & \omega_1 \\ \omega_2 & -\omega_1 & 0 \end{pmatrix} . \tag{21}$$

After some manipulations, equation (19) reduces to

$$D^2 \boldsymbol{V} = -\frac{4}{k^2} (\boldsymbol{\omega} \cdot \boldsymbol{k})^2 \boldsymbol{V} \tag{22}$$

the solution of which is

$$\boldsymbol{V}(\boldsymbol{k}, t) = \mathrm{e}^{-\chi t} \left[\boldsymbol{A}(\boldsymbol{k}) \exp\left\{\frac{2i(\boldsymbol{\omega} \cdot \boldsymbol{k})t}{k}\right\} + \boldsymbol{B}(\boldsymbol{k}) \exp\left\{\frac{-2i(\boldsymbol{\omega} \cdot \boldsymbol{k})t}{k}\right\}\right] \tag{23}$$

with

* These transforms must be regarded as generalized functions (Lighthill, 1960) or as Fourier transforms of hypothetical fields which are zero outside a very large box and otherwise identical to the real fields.

$$\chi = \frac{(k_k b_{ok})^2}{\lambda k^2} + \nu k^2 ,$$

$\boldsymbol{A}(\boldsymbol{k})$ and $\boldsymbol{B}(\boldsymbol{k})$ constant.

The spectral kinetic energy density tensor may be defined either by

$$\Omega_{ij}(\boldsymbol{k}, t) = \tfrac{1}{2} V_i(\boldsymbol{k}, t) \, V_j^*(\boldsymbol{k}, t) \tag{24}$$

or as usually by

$$\Omega_{ij}(\boldsymbol{k}, t) = \int \tfrac{1}{2} \langle v_i(\boldsymbol{x}, t) \, v_j(\boldsymbol{x}+\boldsymbol{r}, t) \rangle \, e^{-i\boldsymbol{k}\boldsymbol{r}} d\boldsymbol{r} \tag{25}$$

where the star denotes a complex conjugate and where $\langle \quad \rangle$ indicates that mean value on all points a distance r apart has been taken at time t. The z-axis being chosen in the direction of the axis of rotation, the kinetic energy may be written

$$\Omega_{ij} = \tfrac{1}{2} e^{-2\chi t} \left\{ A_i A_j^* + B_i B_j^* + A_i B_j^* \exp\left(\frac{4i\omega k_3 t}{k}\right) + A_j^* B_i \exp\left(\frac{-4i\omega k_3 t}{k}\right) \right\} . \tag{26}$$

3. The Initial Conditions for the Kinetic Energy

Idealizing the process of the initial development of homogeneous turbulence, Batchelor and Proudman [3] assume that its large-scale structure is the same as if it had been homogeneous for all time and had developed from an initial stage where all integral moments of cumulants of the velocity distribution converged; they obtain for the initial value of Ω_{ij}:

$$\Omega_{ij}^0 = \tfrac{1}{4} R_{lmnp} \left(\delta_{il} - \frac{k_i k_l}{k^2} \right) \left(\delta_{jm} - \frac{k_j k_m}{k^2} \right) k_n k_p \tag{27}$$

where R_{lmnp} is constant. These results were extended to the case where a magnetic field is present by Alexandrou [4] and Hassan [9].

In his study, Deissler [2] assumed initial isotropy of the turbulence, which leads to the simpler values:

$$\Omega_{ij}^0 = C k^2 \left(\delta_{ij} - \frac{k_i k_j}{k^2} \right) \tag{28}$$

where C is constant.

Though too restrictive in the present work, let us mention that those conditions suppress the ω-dependance of Ω_{ij}.

In 1967, Saffman [8] found new values for the initially anisotropic velocity spectral tensor:

$$\Omega_{ij}^0 = M_{lm} \left(\delta_{il} - \frac{k_i k_l}{k^2} \right) \left(\delta_{jm} - \frac{k_j k_m}{k^2} \right) \tag{29}$$

where M_{lm} is constant. These expressions were obtained by following closely Batchelor and Proudman's work but using a modified and less restrictive basic hypothesis: the convergence concerns the vorticity instead of the velocity. In other words, the field of turbulence is generated initially by random impulsive forces with convergent integral moments of cumulants.

There seems to be no reason why (27) and (29) should not be applied in the case of a fluid in uniform rotation. Indeed the equation for the velocity correlation tensor $R_{ij}(\boldsymbol{r}, t)$ reads:

$$\frac{\partial}{\partial t} R_{ij}(\boldsymbol{r}, t) = \frac{\partial}{\partial r_k} (\overline{u_i u_k u_j'} - \overline{u_i u_k' u_j'}) + \frac{\partial \overline{p u_j'}}{\partial r_i} - \frac{\partial \overline{p' u_j}}{\partial r_j} + 2\nu \nabla^2 R_{ij} + 2\omega_m (\varepsilon_{ilm} R_{lj} + \varepsilon_{jlm} R_{il}) \tag{30}$$

with

$$u_j' = u_j(\boldsymbol{x}') , \quad \boldsymbol{r} = \boldsymbol{x} - \boldsymbol{x}' .$$

Essentially this equation was established by Batchelor and Proudman, except for the last term due to the rotation which is added here. Considering the order of magnitude in r, for large r, of the different terms, Batchelor and Proudman [let us remind that Saffman used the same arguments] proved that, according to their hypothesis, the pressure term was in r^{-5} and so were R_{ij} and $(\partial/\partial t)R_{ij}$, whereas the non-linear term was exponentially small. On this basis, the expressions (27) were derived independently from equation (30). If the fluid is rotating, the basis hypothesis remains unchanged and the order of magnitude of the different terms are unaffected, this being compatible with the additional terms. We have not been able to find any physical reason of an eventual modification of the order of magnitude of R_{ij} due to the Coriolis force influence⋆. Hence we may assume that (27) and (29) still hold for a rotating fluid.

For simplicity the initial conditions (29) established by Saffman are used here. It could be shown easily that they lead to the same results than these of Batchelor and Proudman, but larger of one order in t.

4. Spectral Energy Tensors

As simplifying hypothesis, parallelism of the rotation axis and the applied magnetic field is assumed; then

$$\boldsymbol{b}_o = (0, 0, b_o)\,. \tag{31}$$

To obtain the spectral energy density tensor, each Ω_{ij} is expressed in function of the initial conditions (29) and is integrated over a sphere of radius k:

$$\Omega_{ij}(k, t) = \int_0^{2\pi} d\psi \int_{-1}^{1} \Omega_{ij}(k, \varphi, \psi, t)k^2 d(\cos\varphi)\,. \tag{32}$$

Spherical coordinates are used, φ referring to the axis of rotation.

We consider a non-dimensional time t/θ, where θ is the inhibition time introduced by Nihoul:

$$\theta = \frac{\lambda+\nu}{4b_o^2} \simeq \frac{\lambda}{4b_o^2}\,. \tag{33}$$

Integration over φ⋆⋆ can be done analytically by an asymptotic method—the method of the steepest descent [11]—for large values of t/θ:⋆⋆⋆

$$\frac{t}{\theta} \gg 1\,. \tag{34}$$

After integration over the whole k-space, the following asymptotic forms are obtained for kinetic energy:

$$\Omega_{ij}(t) \sim \alpha\left[C_5^{(ij)} t^{-2} + \sum_{q=0}^{4} C_q^{(ij)} t^{-2}(\omega\theta)^q \mathrm{e}^{-8\omega^2\theta^2(t/\theta)}\right]$$

for $ij = 11, 22, 33, 12, 21$;

$$\begin{aligned}\Omega_{ij}(t) \sim \alpha\{&C_1^{(ij)}(t/\theta)^{-1}t^{-2} + \mathrm{e}^{-8\omega^2\theta^2(t/\theta)}[C_2^{(ij)}t^{-2} + C_3^{(ij)}\omega\theta t^{-2} +\\ &+ C_4^{(ij)}(\omega\theta)^2(t/\theta)^{-1}t^{-2} + C_5^{(ij)}(\omega\theta)^3 t^{-2} + C_6^{(ij)}(\omega\theta)^4 t^{-2}]\}\end{aligned} \tag{35}$$

for $ij = 13, 31, 23, 32$

⋆ G. K. Batchelor (private communication) did confirm this assertion and justifies the inconceivability of a contribution of order larger than r^{-5} by the linearity in the velocity correlation tensor of the term arising from the Coriolis force.

⋆⋆ For the details of those calculations, see C. Frankignoul, 1968, [10].

⋆⋆⋆ Also in the less interesting case $t/\theta \ll 1$, [10].

with

$$\alpha = \frac{\pi^2}{32\nu b_0}\sqrt{\frac{\lambda}{\nu}} \tag{36}$$

and C_q^{ij} constants. Similar expressions are found for the magnetic energy, using equation (16).

5. The Laws of Decay

The total kinetic and magnetic energies decay as t^{-2} and are independent of ω—Coriolis forces are conservative—; their asymptotic values are explicity:

$$E(t) \sim \frac{\pi^2}{16\nu b_o}\sqrt{\frac{\lambda}{\nu}}(M_{11}+M_{22}+2M_{33})t^{-2} \tag{37}$$

$$M(t) \sim \frac{\pi^2}{16\lambda b_o}\sqrt{\frac{\lambda}{\nu}}(M_{11}+M_{22}+2M_{33})t^{-2}\,. \tag{38}$$

This result corresponds to the t^{-3} law obtained by Nihoul [5], [12] in the non-rotating case with Batchelor and Proudman's initial conditions. We have the property:

$$\frac{E(t)}{M(t)} \sim \frac{\lambda}{\nu}, \tag{39}$$

which was found by Nihoul except for a factor 3 due to the different order in k of the initial conditions (27) and (29).

The law of decay of the kinetic and magnetic energy may take two different forms, according to the value of the parameter $8\omega^2\theta^2(t/\theta)$. In the case

$$8\omega^2\theta^2\frac{t}{\theta} \ll 1 \tag{40}$$

the asymptotic form of the kinetic energy tensor is:

$$\Omega(t) \sim \alpha\begin{pmatrix} \frac{1}{2}(3M_{11}+M_{22})t^{-2} & M_{12}t^{-2} & 2M_{13}t^{-2} \\ M_{12}t^{-2} & \frac{1}{2}(M_{11}+3M_{22})t^{-2} & 2M_{23}t^{-2} \\ 2M_{13}t^{-2} & 2M_{23}t^{-2} & 4M_{33}t^{-2} \end{pmatrix} \tag{41}$$

The magnetic energy tensor has the same value, with β instead of α:

$$\beta = \frac{\pi^2}{32\lambda b_o}\sqrt{\frac{\lambda}{\nu}}\,. \tag{42}$$

Comparing the energy in the rotation-axis and $\boldsymbol{B}$ direction with the total energy, it is found that:

$$\frac{\Omega_{33}(t)}{\frac{1}{3}E(t)} \sim \frac{6M_{33}}{M_{11}+M_{22}+2M_{33}} \tag{43}$$

which is an analytical confirmation of the value $\frac{3}{2}$, Deissler [2] observed on his computed curves in the case of non-rotating turbulence with initial isotropy. (Then $M_{11}=M_{22}=M_{33}$). We may conclude that the laws of decay are not modified by a uniform rotation for small values of $8\omega^2\theta^2(t/\theta)$.

In the case

$$8\omega^2\theta^2\frac{t}{\theta} \gg 1 \tag{44}$$

the kinetic energy tensor takes the asymptotic form:

$$Q(t) \sim \alpha \begin{pmatrix} \frac{1}{4}(3M_{11}+M_{22}+4M_{33})t^{-2} & \frac{1}{2}M_{12}t^{-2} & (\lambda M_{13}/2b_o^2)t^{-3} \\ \frac{1}{2}M_{12}t^{-2} & \frac{1}{4}(M_{11}+3M_{22}+4M_{33})t^{-2} & (\lambda M_{23}/2b_o^2)t^{-3} \\ (\lambda M_{13}/2b_o^2)t^{-3} & (\lambda M_{23}/2b_o^2)t^{-3} & (M_{11}+M_{22}+2M_{33})t^{-2} \end{pmatrix} \tag{45}$$

We have the relation:

$$\Omega_{11}+\Omega_{22}=\Omega_{33}\,. \tag{46}$$

The same expression still holds for magnetic energy, with β instead of α.

The main feature of this second case is that the exchange terms between energy along $\boldsymbol{\omega}$ and $\boldsymbol{B}$ and energy in the planes perpendicular to that direction decay faster by one order in t and consequently become rapidly negligible, leading to independent behaviour of these energies though they always remain equal, as it is seen by (46). We are in presence of a plane turbulent motion, modulated perpendicularly with the same total energy. The form of the energy tensors

$$\begin{pmatrix} \mathrm{X} & \mathrm{X} & \mathrm{O} \\ \mathrm{X} & \mathrm{X} & \mathrm{O} \\ \mathrm{O} & \mathrm{O} & \mathrm{X} \end{pmatrix} \tag{47}$$

combined with (46), suggest a striking analogy with the inertial tensor of a flat body rotating around an axis perpendicular to its plane of symmetry.

The results (45) are in agreement with predictions of Lehnert [1] who established that the time dependence of the vorticity tensor elements with at least one indice in the rotation axis direction was not influenced by the Coriolis force. As the vorticity is the curl of the velocity, this corresponds to Ω_{11}, Ω_{22}, Ω_{12}. We might expect that the modulated plane turbulent motion becomes a ordinary plane turbulent motion when $\boldsymbol{\omega}$ and $\boldsymbol{B}$ are not parallel (which was not imposed by Lehnert) with Ω_{33} decreasing then as t^{-3}. Let us finally notice that an eventual initial anisotropy of energy has less influence in the second stage than in the first.

The two successive asymptotic regimes are separated by a short transition period $\sim O(10^2$ sec). With a suitable choice of the parameters, for instance usual values for mercury experiments like:

$$b_0 \sim O(10) \qquad \theta \sim O(10^{-2}) \qquad \omega \sim O(1) \tag{48}$$

both successive stages could be observed experimentally. Measurements would be much simpler in the second stage: the energy has just to be known in one direction to have the ratio (39); this would give additional information to decide between Batchelor's and Saffman's hypotheses.

6. Conclusions

The last period of decay of the kinetic and magnetic energy of homogeneous turbulence in a weakly conducting fluid in uniform rotation is characterized by two different asymptotic behaviours which may occur successively. In the first regime, the rotation does not modify the time dependence of all energy tensor components which remains in t^{-2} with Saffman's initial conditions (t^{-3} with Batchelor and Proudman's one). In the second regime which is strongly anisotropic, the rotation gives rise to independent behaviour of the energy along the axis of rotation and the applied magnetic field, assumed to be parallel, and the energy in the perpendicular planes though these two contributions remain equal. The modulation of the plane turbulent motion arises from the faster decay, in t^{-3} (t^{-4} with (27)), of the corresponding exchange terms and seems to be characteristic of the parallelism of ω and B.

Our results suggest an analogy between the effect of rotation and stratification as observed in non-turbulent fluids (e.g. [13]).

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